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Extension of the Gurney Equations to Two Dimensions for a Cylindrical Charge

by Benjamin A. Breech

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14. ABSTRACT We provide a two-dimensional extension to the traditional Gurney equations describing the velocity of fragments from an exploding charge. Our approach differs from the recent Tie-peng et al. publication by applying different underlying assumptions that may be more physically realistic. We demonstrate the procedure by computing the fragment velocity from an detonating cylindrical charge surrounded by a thin shell with metal plates on top and bottom. We also demonstrate that under specific conditions, the the results of Tie-peng et al. 's approach and ours are identical.			
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Contents

List of Figures

1. Introduction

The Gurney equations (*1*) provide an estimation of the velocities of fragments from an detonating charge. The equations derive from the principles conservation of energy and momentum applied to the gases produced by the detonation and the shell surrounding the charge. The equations assume the fragments travel in one dimension (e.g., radially in the case of a cylinder).

Tie-peng et al. (*2*) recently demonstrated an extension to the Gurney equations for the case of a cylindrical charge surrounded on the sides by a metal shell and with metal plates on top and bottom. The Tie-peng et al. result assumes the detonation wave reaches the top and the surrounding shell at the same time regardless of the radius and height of the cylinder. This necessitates the assumption that the acceleration of the gases due to the explosion is not uniform in all directions.

In this work, we adopt a different approach by assuming the acceleration of the gas due to the explosion is uniform. We then work out a different version of the Gurney equations for the cylindrical charge described. The resulting equations may be simpler to use than the Tie-peng et al. as they do not rely upon quantities that are difficult to measure, such as the pressure on the plates and sides.

Section 2 provides some background material on the Gurney equations. Section 3 then describes the extension to the equations. We finish by providing some concluding remarks in section 4.

2. Background: Derivation of the Gurney Equations

This section provides background material on the Gurney equations. In particular, we show the derivation for specific geometries of interest. Readers who are already familar with the Gurney equations may skip this section.

The Gurney equations are based on some simple assumptions and modeling. Initiating the charge results in a detonation wave that moves outward generating gaseous combustion products that flow along behind it. Upon contact with the casing, the gases (e.g., combustion products) force the casing to fragment and then push the fragmets outward with some velocity. The Gurney equations ignore the work done on the casing to cause the fragmentation. Instead, the casing (and all fragments) are assumed to go outward at the same velocity as the gases when contact was made.

We obtain the velocity of the fragments by applying conservation of energy and, where applicable, conservation of momentum to the gases and casings. Importantly, we assume that all of the chemical potential energy in the explosion goes into the kinetic energy of the fragments and combustion products. This ignores the work done on the casing to cause fragmentation, as well as the increased internal energy of the explosive, but allows for a closed form estimate.

2.1 Cylindrical Charge

Let us now consider a simple cylindrical charge with radius R surrounded on the sides by a thin metal casing. The charge has a mass per unit length of C. The casing has a mass per unit length of M_s. Figure 1 shows the example charge.

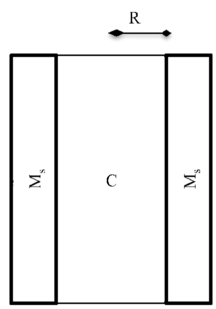

Figure 1. Diagram of a cylindrical explosive charge surrounded by a thin shell of mass M_s.

To simplify the geometry, we require the detonation wave to start uniformly along the center line of the cylinder (e.g., $r = 0$) at the same time. The detonation wave will then expand outward in all directions, enforcing azimuthal symmetry. We make the additional assumption that the detonation wave has no significant effects in the $\hat{z}$ direction. In section 3, we show how to relax this assumption. With these simplifications, the problem reduces to the radial direction only.

To find the velocity of the fragments, we enforce conservation of momentum and energy. Due to the azimuthal symmetry, conservation of momentum provides no useful information as momentum is trivially conserved. We thus turn our attention to conservation of energy by computing the total kinetic energy and equating to the chemical energy stored in the explosive.

The blast wave accelerates gases outward. We take the velocity of the gases when they reach the shell to be v_s. The blast then fragments the shell and pushes the fragments outward. We will assume the fragments move at the same velocity as the gases, i.e., v_s. The total kinetic energy of the fragments is then $(1/2)M_s v_s^2$.

The gases also have kinetic energy, which must be accounted for. The blast begins at the center axis, where the velocity of the gases would be 0. The gases reach a maximum velocity of v_s at the shell. We therefore assume the velocity of the gases varies linearly with position, e.g.,

$$v(r) = \frac{r}{R} v_s \tag{1}$$

where R is the radius of the cylinder. Taking $\rho = C/\pi R^2$ to be the density of the charge, we can compute the kinetic energy (per unit length) of the charge products, T_c, as

$$\begin{aligned} T_c &= \frac{1}{2}\int_0^R 2\pi\rho r v^2(r)\,dr \\ &= \frac{1}{2}2\pi\rho v_s^2 \int_0^R \frac{r^3}{R^2}\,dr \\ T_c &= \frac{1}{2}\frac{C}{2}v_s^2 \end{aligned} \tag{2}$$

Let E denote the chemical energy per unit mass for the particular explosive being used. The total chemical energy per unit length of the explosive is given by EC. Following Gurney (*1*), we assume the total chemical energy available in the explosive converts entirely to kinetic energy. This provides us with the required relation to determine v_s,

$$\begin{aligned} EC &= \frac{1}{2}M_s v_s^2 + \frac{1}{2}\frac{C}{2}v_s^2 \\ 2E &= v_s^2\left(\frac{M_s}{C} + \frac{1}{2}\right) \end{aligned}$$

$$v_s = \sqrt{2E}\sqrt{\alpha} \qquad \alpha^{-1} \equiv \frac{M_s}{C} + \frac{1}{2} \tag{3}$$

The constant $\sqrt{2E}$, which has units of velocity, is commonly referred to as Gurney's Constant and varies with the explosive used. All of the Gurney equations take the form of equation 3. The equations differ only with the definition of α.

If we substantially increase M_s, then equation 3 predicts the fragments would move outward with smaller velocity, which agrees with intuition. Interestingly, equation 3 predicts an asymptotic maximum speed of $\sqrt{2}\sqrt{2E}$ for fragments as the amount of explosive is increased.

2.2 Sandwich Charge

Another useful example is a sandwich charge, where two flat plates, with masses per unit area of M_1 and M_2, are separated by a charge mass per unit area C. If $M_1 = M_2$, then the entire package is referred to as a symmetric sandwich charge. If the two masses are not equal, the package is a asymmetric sandwich charge. The general case is shown in figure 2.

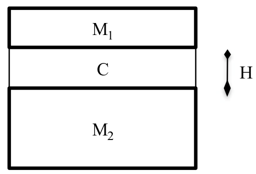

Figure 2. Sandwich charge configuration. If $M_1 = M_2$, the charge is referred to as a 'symmetric sandwich charge'. Otherwise, the charge is an 'asymmetric sandwich charge' .

For convenience, we orient the charge and plates so that the blast would send the plates in the vertical ("$\hat{y}$") direction. Again, we reduce the problem to one dimension by assuming the velocity of gaseous combustion products are insignificant in the $\hat{x}$ direction.

As the blast wave goes outward, the gases imparts a velocity v_1 to the M_1 plate and a velocity $-v_2$ to the M_2 plate (minus sign added because the M_2 plate will move in the opposite direction of M_1). We assume the velocity of the explosion products varies linearly in the $\hat{y}$ direction. We thus have

$$v(y) = (v_1 + v_2)\frac{y}{H} - v_2 \tag{4}$$

where H is the height of the charge.

The problem is to determine the velocities, v_1 and v_2. As before, we apply conservation of momentum and energy to obtain the required relations

From conservation of momentum, we have

$$0 = M_1 v_1 - M_2 v_2 + \int_0^H \rho v(y)\, dy, \tag{5}$$

where $\rho = C/H$ is the mass density. From this, we obtain the first relation,

$$v_2 = Av_1 \qquad A \equiv \frac{1+2(M_1/C)}{1+2(M_2/C)}. \tag{6}$$

The kinetic energy relation is

$$EC = \frac{1}{2}M_1 v_1^2 + \frac{1}{2}M_2 v_2^2 + \frac{1}{2}\int_0^H \rho v^2(y)\,dy \tag{7}$$

Carrying out the computation, using equation 6 and noting that $1 - A + A^2 = (A^3+1)/(A+1)$, we arrive at the Gurney equation,

$$\begin{aligned} v_1 &= \sqrt{2E}\sqrt{\alpha_1} \\ \alpha_1^{-1} &\equiv \frac{M_1}{C} + A^2\frac{M_2}{C} + \frac{1}{3}\frac{A^3+1}{A+1} \end{aligned} \tag{8}$$

We can compute some interesting limits from equation 8. If the masses of the plates are equal, then we obtain the symmetric sandwich equation with

$$\alpha_1^{-1} = 2\frac{M_1}{C} + \frac{1}{3}. \tag{9}$$

Another interesting limit occurs when one of the plates is not present (the "open face" sandwich). For convenience, we pick $M_2 = 0$ so there is no bottom plate. In this case, v_2 then becomes the velocity gas exiting from below the charge. We obtain

$$\alpha_1^{-1} = \frac{(1+2\frac{M_1}{C})^3+1}{6(1+\frac{M_1}{C})} + \frac{M_1}{C} \tag{10}$$

The last limit of interest occurs when we allow one of the plates, say M_2, to have substantially large mass, e.g., $M_2 \to \infty$. If we apply this limit to equation 8, we find that $v_1 = \sqrt{2E}\sqrt{\alpha_1}$ with

$$\alpha_1^{-1} = \frac{M_1}{C} + \frac{1}{3} \tag{11}$$

The result is correct, but only by coincidence as there is a subtle inconsistency. The problem lies in applying conservation of momentum. For a plate of infinite mass, the bottom plate does not move ($v_2 = 0$), nor do gases from the explosion escape from the bottom. As a result, conservation of momentum, equation 5, would imply that $v_1 = 0$ for this case, which is inconsistent with equation 11. Since the bottom plate is not moving, momentum is not conserved which gives rise to this inconsistency (the bottom plate basically provides an external force on the system). The proper way of handling this limit is to take $v_2 = 0$ in the kinetic energy relation, equation 7. We would then obtain equation 11.

2.3 Limitations and Discussion

Generally, the Gurney equations consider one dimension only by considering “ideal” charge configurations. The “ideal” configurations impose some symmetry condition and have infinite length allowing edge effects to be ignored. Some of these limitations can be relaxed. Edge effects can be accounted for by adjusting the M/C ratio (*3*) or by introducing correction terms (*4*). Symmetry conditions could be relaxed, but at the expense of dramatically increased complexity.

All of the Gurney equations depend on the constant $\sqrt{2E}$, which must be measured for each explosive. $\sqrt{2E}$ can be measured either by empirically fitting the Gurney equations or through calormetric means (*5*). The computated velocities obviously depend on the quality of the measurement of the constant.

The main advantage of the Gurney equations lie in their simplicity. The equations can quickly be derived and can provide good estimates of the fragment velocities. There may be compensating errors present in the analysis. The energy required to fragment the casing is ignored, which should cause the equations to over estimate the fragment velocities. On the other hand, using simple velocity profiles that vary linearly can underestimate the actual velocities of the combustion products. Actual detonations cause shocks to develop which will cause combustion products to move faster than a simple linear fit.

3. Extension to the Gurney Equations

Here we discuss our extension to two dimensions for the cylindrical charge described in section 1. The geometry of the problem incorporates both the cylinder and the asymmetric sandwich problems discussed in section 2.

We now move to two dimensions. The charge configuration is shown in figure 3. The charge has a radius of R, and height of H. At $z = 0$, we attach a plate of mass M_2. A second plate of mass M_1 is attached at $z = H$. A metal shell of mass M_s surrounds the charge.

For this case, we assume the detonation point is located along the center axis ($r = 0$). This provides azimuthal symmetry. The height of the detonation point is determined by velocity profile imposed on the combustion products.

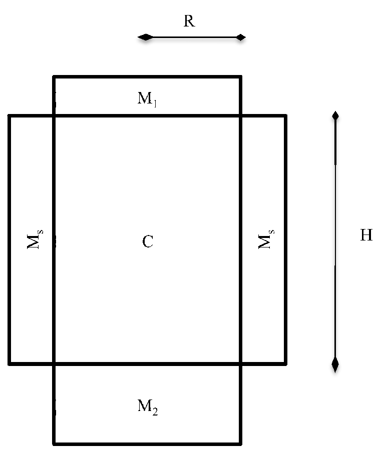

Figure 3. Cross section of the cylindrical charge considered for this work. The charge has a radius of R and height of H. Two plates with masses M_1 and M_2 attach to the top and bottom of the charge. Additionally, the charge is surrounded by a metal shell with mass M_s .

Our problem is to compute the velocity of fragments, v_1, v_2, and v_s, created from the plates and shell given the masses of the plates, M_1 and M_2, the mass of the shell, M_s, the mass of the explosive, C, and the Gurney Constant for the explosive, $\sqrt{2E}$.

As before, we assume the blast pushes gases outward with a velocity that varies linearly. In the radial direction, the velocity varies with distance from the center line. Along the height of the cylinder, the velocity varies linearly between $-v_2$ and v_1. Combining these, we have

$$\mathbf{v}(r,z) = \frac{r}{R} v_s \hat{r} + \left[(v_1 + v_2)\frac{z}{H} - v_2 \right] \hat{z} \tag{12}$$

With this velocity profile, we can examine conservation of momentum. In the radial direction, momentum is trivially conserved and provides no useful relations. In the $\hat{z}$ direction, however, we obtain

$$0 = M_1 v_1 - M_2 v_2 + \int_{r=0}^{R} \int_{z=0}^{H} 2\pi\rho r \left[(v_1 + v_2)\frac{z}{H} - v_2 \right] dr\, dz$$

which gives our first relation,

$$v_2 = A v_1 \qquad A \equiv \frac{1 + 2(M_1/C)}{1 + 2(M_2/C)} \tag{13}$$

The kinetic energy relation is

$$CE = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_2 v_2^2 + \frac{1}{2} M_s v_s^2 + \frac{1}{2} \int_{r=0}^{R} \int_{z=0}^{H} 2\pi\rho r |\mathbf{v}(r,z)|^2 \, dr\, dz \tag{14}$$

which, combined with equation 13, gives the second relation among the velocities,

$$2E = \left(\frac{M_1}{C} + A^2 \frac{M_2}{C} + \frac{1}{3} \frac{A^3 + 1}{A + 1} \right) v_1^2 + \left(\frac{M_s}{C} + \frac{1}{2} \right) v_s^2 \tag{15}$$

We clearly see that the first term on the right hand side of equation 15 corresponds to the Gurney equation for an asymmetric sandwich, while the second term corresponds to the cylinder charge. If we take $M_s \to \infty$, which requires $v_s \to 0$, then we obtain the Gurney equation for the asymmetrical sandwich. If we repeat the process for the limits of $M_1, M_2 \to \infty$, we recover the Gurney equation for the cylindrical charge.

If we empirically know either v_1 or v_s, then equation 15 may be used to obtain the other. However, we can make no further analytic progress without additional assumptions.

3.1 Relating v_1 to v_s

One approach to relate v_1 and v_s is to assume the detonation wave accelerates the gases uniformly in all directions. The gases will, in general, reach the shell and the plates at different times. If a is the acceleration, we can compute the velocity and distances the gases travel as

$$v_s = at_s \quad \text{and} \quad R = \frac{1}{2}at_s^2 \tag{16}$$

for the radial direction. This implies

$$a = \frac{1}{2}\frac{v_s^2}{R}. \tag{17}$$

In the axial direction, the distance traveled is dependent on the location of the detonation point. For symmetry reasons, we assumed the detonation point was located along the $r = 0$ line. We can locate the axial location by computing where $v_z = 0$ from the velocity profile given in equation 12,

$$v_z(z_0) = 0 = (v_1 + v_2)\frac{z_0}{H} - v_2 \tag{18}$$

$$z_0 = \frac{v_2}{v_1 + v_2}H = \frac{A}{1+A}H \tag{19}$$

where equation 13 was used.

The gases cover a distance of $H - z_0$, which can then be used to write down

$$v_1 = at_1 \quad \text{and} \quad \frac{1}{1+A}H = \frac{1}{2}at_1^2 \tag{20}$$

which implies

$$a = \frac{1+A}{2H}v_1^2. \tag{21}$$

Equating equations 17 and 21, we obtain the needed relation between v_1 and v_s,

$$v_1^2 = \frac{H}{R}\frac{1}{A+1}v_s^2. \tag{22}$$

Substituting equation 22 back into equation 15 provides us the equation for v_s,

$$v_s = \sqrt{2E}\sqrt{\alpha_s} \qquad \alpha_s^{-1} \equiv \frac{1}{A+1}\left(\frac{M_1}{C} + A^2\frac{M_2}{C} + \frac{1}{3}\frac{A^3+1}{A+1}\right)\frac{H}{R} + \left(\frac{M_s}{C} + \frac{1}{2}\right) \tag{23}$$

Using this with equations 13 and 22 gives the other velocities,

$$v_1 = \sqrt{2E}\sqrt{\alpha_1} \quad \alpha_1^{-1} \equiv \left(\frac{M_1}{C} + A^2\frac{M_2}{C} + \frac{1}{3}\frac{A^3+1}{A+1}\right) + (A+1)\left(\frac{M_s}{C} + \frac{1}{2}\right)\frac{R}{H} \tag{24}$$

$$v_2 = \sqrt{2E}\sqrt{\alpha_2} \quad \alpha_2^{-1} \equiv \frac{1}{A^2}\left(\frac{M_1}{C} + A^2\frac{M_2}{C} + \frac{1}{3}\frac{A^3+1}{A+1}\right) + \frac{A+1}{A^2}\left(\frac{M_s}{C} + \frac{1}{2}\right)\frac{R}{H} \tag{25}$$

We may take limits of these equations, but doing so requires careful interpretation. All the speeds depend, in some fashion, on the height to radius ratio, H/R. When $H \ll R$, the problem geometry becomes two plates sandwiched around a layer of charge. We may expect that the equations would then reduce to the sandwich equations shown in section 2. However, equations 23, 24, and 25 emphasize the cylindrical charge equation instead. The reason for the seemingly contradictory limit lies in the assumptions used to obtain a relationship between v_1 and v_s. We assumed that the blast wave accelerates the gases uniformly. If $H \ll R$, then the gases in the $\hat{z}$ direction have far less time to be accelerated before they encounter the plates than the gases in the radial direction. As a result, most of the available energy will go into the fragments from the shell. Hence, the cylindrical charge should be emphasized. When considering limits of this type, it may be better to apply the limits to the kinetic energy relation shown in equation 15.

3.2 Tie-peng et al. Method

Recently, Tie-peng et al. (*2*) examined this identical charge configuration, but with different underlying assumptions. In particular, they obtain relations between v_1 and v_s by assuming the detonation wave reaches the top plate and the surrounding shell *at the same time*. This necessarily implies the acceleration of the gases due to the blast wave is not uniform in all directions, but rather accelerates the gases differently in the $\hat{r}$ and $\hat{z}$ directions.

We can obtain the accelerations by introducing the pressures on the top plate, P_1, and the surrounding shell, P_s. The definition of pressure immediately provide

$$P_1 = \frac{M_1 a_1}{\pi R^2} \qquad P_s = \frac{M_s a_s}{2\pi R H} \tag{26}$$

Since Tie-peng et al. assumed the detonation wave reaches the plate and shell at the same time, we can immediately write

$$\frac{v_1}{v_s} = \frac{a_1}{a_s} = \frac{M_s}{M_1}\frac{P_1}{P_s}\frac{R}{2H} \tag{27}$$

Using this relation in equation 15, we obtain the Tie-peng et al. result,

$$\begin{aligned} v_1 &= \sqrt{2E}\sqrt{\alpha_t} \\ \alpha_t^{-1} &= \frac{1}{3}\frac{A^3+1}{A+1} + A^2\frac{M_2}{C} + \frac{M_1}{C}\left[16\frac{P_s^2}{P_1^2}\frac{M_1}{M_s}\frac{H^2}{d^2}+1\right] + 8\frac{P_s^2}{P_1^2}\frac{M_1^2}{M_s^2}\frac{H^2}{d^2}, \end{aligned} \tag{28}$$

where $d = 2R$ is the diameter of the cylinder.

The Tie-peng et al. approach and our approach utilize different underlying assumptions. However, we note that when $H = 2R$, and $M_1 = M_2$, the two methods should produce the same result. $M_1 = M_2$ implies that $v_1 = v_2$, and therefore the detonation point is at the center of the cylinder. In the Tie-peng et al. procedure, $H = 2R$ implies that the acceleration must be uniform in all directions in order for the blast wave to hit the top and side at the same time. Within our approach, the blast wave also hits the top plate and side shell at the same time for this configuration.

When $H = 2R$ (and recalling that, for this case, $a_s = a_1$), we immediately find from equation 26,

$$\frac{P_s}{P_1} = \frac{M_s}{4M_1} \tag{29}$$

Due to the assumption that $M_1 = M_2$, we must also have that $A = 1$.

Putting all this into equation 28, we find

$$\begin{aligned} \alpha_t^{-1} &= \frac{1}{3}\frac{A^3+1}{A+1} + A^2\frac{M_2}{C} + \frac{M_1}{C}\left[16\left(\frac{M_s}{4M_1}\right)^2\frac{M_1}{M_s}\frac{H^2}{d^2}+1\right] + 8\left(\frac{M_s}{4M_1}\right)^2\frac{M_1^2}{M_s^2}\frac{H^2}{d^2} \\ &= \left(\frac{1}{3}+\frac{M_2}{C}+\frac{M_1}{C}\right) + \left(\frac{M_s}{C}+\frac{1}{2}\right). \end{aligned} \tag{30}$$

which is identical to our result shown in equation 24 when $H = 2R$ and $A = 1$.

3.3 Discussion of Differences and Limitations

The differences in the two procedures lie in how the third and final relation among the velocities is determined. The Tie-peng et al. result assumes the blast wave accelerates gases in such a way as to make the gases reach the top plate and the surrounding shell at the same time. This may not be physically realistic as the blast wave should emanate from the detonation outward causing an acceleration of the gases that is the same in all directions. The assumption of the same acceleration in all directions underlies our approach. Furthermore, our approach may be simpler to use as it does not rely upon the pressures on the plate and shell, which are difficult to measure quantities.

At the same time, however, we note that both procedures ignore more complicated physics that can happen when considering two dimensional flows from a detonation. In particular, once the gases impact on the surface closest to the detonation point, the surface will cause some of the gases to reflect back, or deflect into the other direction. Such a process would invalidate the assumptions of both procedures. A more thorough investigation of the complex physics involved is beyond the scope of this work.

Another limitation to note is that neither method handles cases where the radius is “much different” than the height of the charge. For example, let us consider a charge whose height is four times the radius. We (somewhat) arbitrarily place the detonation point $r = 0, z = R$ (e.g., one quarter of the height). As the blast proceeds, the detonation wave will “fill” the bottom half of the cylinder. That is, the detonation wave will reach the bottom plate and the lower half of the surrounding metal shell at the same time. However, the top portion will remain unaffected at this time. Effectively, the bottom portion of the charge will explode well before the top portion even recognizes a detonation has occured. This case is not accounted for in the Gurney style methods and would likely require the use of hydrocodes for a proper analysis.

The most applicable cylindrical charge configuration for this method is one where the height is twice the radius, a detonation point located at the center (e.g., $r = 0, z = H/2$) and whose top and bottom plates have the same mass. This configuration should avoid the limitations discussed in the previous paragraph. The velocities of the fragments will then be given by equations 23, 24, and 25. The velocity estimates may also be applicable when H is not “too different” from $2R$. That is we may expect the velocity estimates given above will still be reasonable even if H is slightly larger than $2R$. We feel that the estimates will certainly break down once H gets near to $4R$. Similarly, the estimates may hold if H is decreased to values less than $2R$, but will break down again at some lower limit (perhaps $H = R$). Comparisons with hydrocodes would be useful to determine the exact range of applicability.

4. Conclusion

In this work, we examined extensions to the Gurney equations for fragments that can travel in two dimensions. Our work was based somewhat in the work of Tie-peng et al. , but with different underlying assumptions. In particular, we assumed that the blast wave causes an accleration that is the same in all directions. This results in a simple equation for the velocity fragments that bears similarities to the usual cylindrical and sandwich Gurney equations.

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